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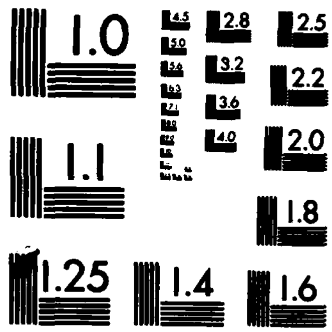

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UNIVERSITY OF SOUTHERN CALIFORNIA

Research Report 80-2

EQUAL WEIGHTS, FLAT MAXIMA, AND TRIVIAL DECISIONS

Richard S. John, Ward Edwards, and Detlof v. Winterfeldt

Social Science Research Institute
University of Southern California

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School of Business
University of Kansas

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SUMMARY

Egon Brunswik first called attention to the importance of ecological validity, and consequently to the importance of correlations among cues. Cue intercorrelations and value independence are central to weights in linear models used for prediction. Dawes, Einhorn and Wainer have argued that equal weights are often better than least-squares regression weights. Newman, Seaver, Edwards and McClelland have all shown that if cue intercorrelations are positive or zero, equal weighting can lead to serious errors of prediction.

Most predictions are intended as a basis for decision making. The point of this paper is that prediction and decision require different methods. Equal weights, while often useful for prediction, are less useful for decision making.

The action options available in any decision problem fall into three classes: sure winners, sure losers, and contenders. Sure winners and sure losers are defined by dominance, accepting sure winners and rejecting sure losers is trivial. Good decision rules should discriminate well among contenders.

In the familiar pick-1 decision problem, options on the Pareto frontier (i.e. undominated options) almost always show negative correlations among attributes. Such negative correlations make equal weights inappropriate.

This paper extends that result to the case in which a decision maker must pick k options out of n. In this case, the set of sure winners is usually not empty. It develops general procedures for identifying the set of contenders, given the options, k, and n.

This set is a generalized Pareto frontier, of which the traditional kind is a special case. Simulations show that attribute intercorrelations among contenders are substantially depressed and typically negative, even if the intercorrelations in the whole set are positive. Such negative correlations among contenders strongly question the usefulness of equal weights for decision making.

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ACKNOWLEDGEMENTS

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I. Introduction

Since Wilks (1938) first published on the robustness of equal weights, many have argued that the weighting question is trivial. Indeed, in psychometrics, differential weighitng of component scores of a test battery is rare. Formal analytic work demonstrating the excellent correspondence between different sets of composites derived from different weight schemes (Ghiselli, 1964; Gulliksen, 1950) abounds in the mental tests literature.

This wheel has been rediscovered many times, most recently in the area of human judgment and decision making and multiple linear regression. There is now little doubt that when dimensions are positively correlated, virtually any weighting scheme is acceptable (Dawes & Corrigan, 1974; Einhorn & Hogarth, 1975; Newman, 1977; Wainer, 1976, 1978). Given agreeable (i.e., non-negative) intercorrelation matrices, it hardly matters whether the weights are obtained subjectively, statistically, randomly, or <u>a priori</u> (i.e., equal weights); the results are essentially the same.

Recent arguments from both the human judgment and regression analysis literatures have challenged the "non-negative intercorrelation assumption." Calling attention to the importance of suppressor variables in multiple regression, Keren and Newman (1978) rejected the equal weighting approach as a general methodology. Negative correlations are present, by definition, in the case of suppressor variables; thus, the one assumption critical to the unit weighting argument is simply not met in at least this one important case of linear regression.

In the area of human judgment and decision making, the non-negative intercorrelation assumption is even more tenuous. Research in multiattribute utility measurement (MAUM) by Edwards and his associates (Edwards, 1976; Newman, Seaver, & Edwards, Note 1; Seaver Note 2), and by McClelland (Note 3) showed that attributes will be negatively correlated if the domain of available acts is restricted to those on the Pareto frontier. (The Pareto frontier of any set of alternatives consists of those that are not dominated. Although dominance may be defined in many ways, an ordinally dominated act is one that is no better than some other act on each dimension and worse than it on at least one dimension.) Of course, for the task of either describing or prescribing choice behavior, only those alternatives on the Pareto frontier are of interest. By adding various dominated (irrelevant) acts one could generate <u>any</u> intercorrelation matrix. However, if an act has no chance of being chosen (which is the case for dominated acts), why consider it at all?

Working only with acts on the Pareto frontier, McClelland (Note 3) Newman et al., (Note 1), and Seaver (Note 2), concluded that composites derived from unit weighting will not agree satisfactorily with those obtained from differential weights. In addition, McClelland (Note 3) showed that the overall value of the best composite determined from unit weighting may be substantially less than that obtained from the correct differential weights, where overall value is computed using the "true" differential weights. These analytic results suggest that the equal weighting argument is not applicable to the multiattribute decision problem.

The intellectual confusion underlying the equal weights argument in decision making arises from using regression, a predictive device, for decision making. Like regression, multiattribute utility measurement (MAUM) combines numbers into composites, usually by rescaling them and then taking a weighted (compensatory) combination, usually a weighted average. It differs from regression in that the numbers and weights are often judged rather than recovered from data and that the method of rescaling is different. It also differs from regression in that its explicit goal differs from the explicit goal of regression, though in fact the two procedures frequently serve the same purpose. A regression coefficient, however calculated, is in fact a descriptive statistic, intended primarily for subsequent use as a tool for prediction. It seems extremely natural and appropriate to base decisions on the predicted characteristics of the options (e.g., candidates for admission to graduate school) being considered. But doing so raises a new set of problems, quite unrelated to the mathematics on which the prediction mechanism was based. Some of these are examined below.

<u>The triviality of many decisions</u>. It is both a fact of experience and a fact of the mathematics of decision theory that most decisions are trivial. The point applies everywhere, but is nicely illustrated in the context, standard for discussions of equal weights, of graduate student admissions. Anyone who has ever served on an admissions committee knows that some small fraction of the admissions folders require essentially no discussion before acceptance; the only question is whether the student will in fact come, since one can be confident that there will be competing admissions from other universities. An even larger fraction of those folders require essentially

no discussion before rejection; in fact, most departments make elements of the rejection process automatic by setting prior standards concerning Graduate Record Examination Scores, grade point averages, or both. The result is that 80% to 90% of the discussion time is devoted to marginal cases. Ignoring for the moment the practical problems of financing students and of match-up between student and faculty interests, a marginal case is marginal for one or the other (or both) of two reasons. Either various indicators disagree, in which case one is unsure what to believe, or all indicators are close to borderline.

Evaluating decisions and evaluating decision rules. Most of the discussions of equal weights, like most other discussions of decisions, evaluate the resulting decisions--the trivial along with the difficult. Thus, evaluation of graduate student admissions, if done at all, may be done by comparing graduate school grade point average (GPA) with undergraduate GPA. Obviously, this comparison will include all students admitted, whether the decision to admit them was hard or easy.

Exactly this kind of evaluation underlies the argument for equal weights. Advocates quite correctly point out that, by virtually any criterion you might wish to specify, the difference defined over the whole set of objects of evaluation between the merits of a selected subset based on equal weights and those based on weights obtained in other ways is often negligible. (Equal weights have collateral advantages, such as independence of data and consequent robustness, but they are irrelevant to this discussion.)

There is a pragmatic virtue in the process of evaluating the whole set of objects of choice selected by a decision process. But it ignores the point that many inclusions and exclusions were not in fact in dispute; all decision processes under consideration would have produced them.

That leads us to the crucial issue of this paper. Are choices left unaffected by whatever differences may exist between decision rule A and decision rule B relevant to the comparative evaluation of those rules? To us, it seems apparent that the answer should be no.

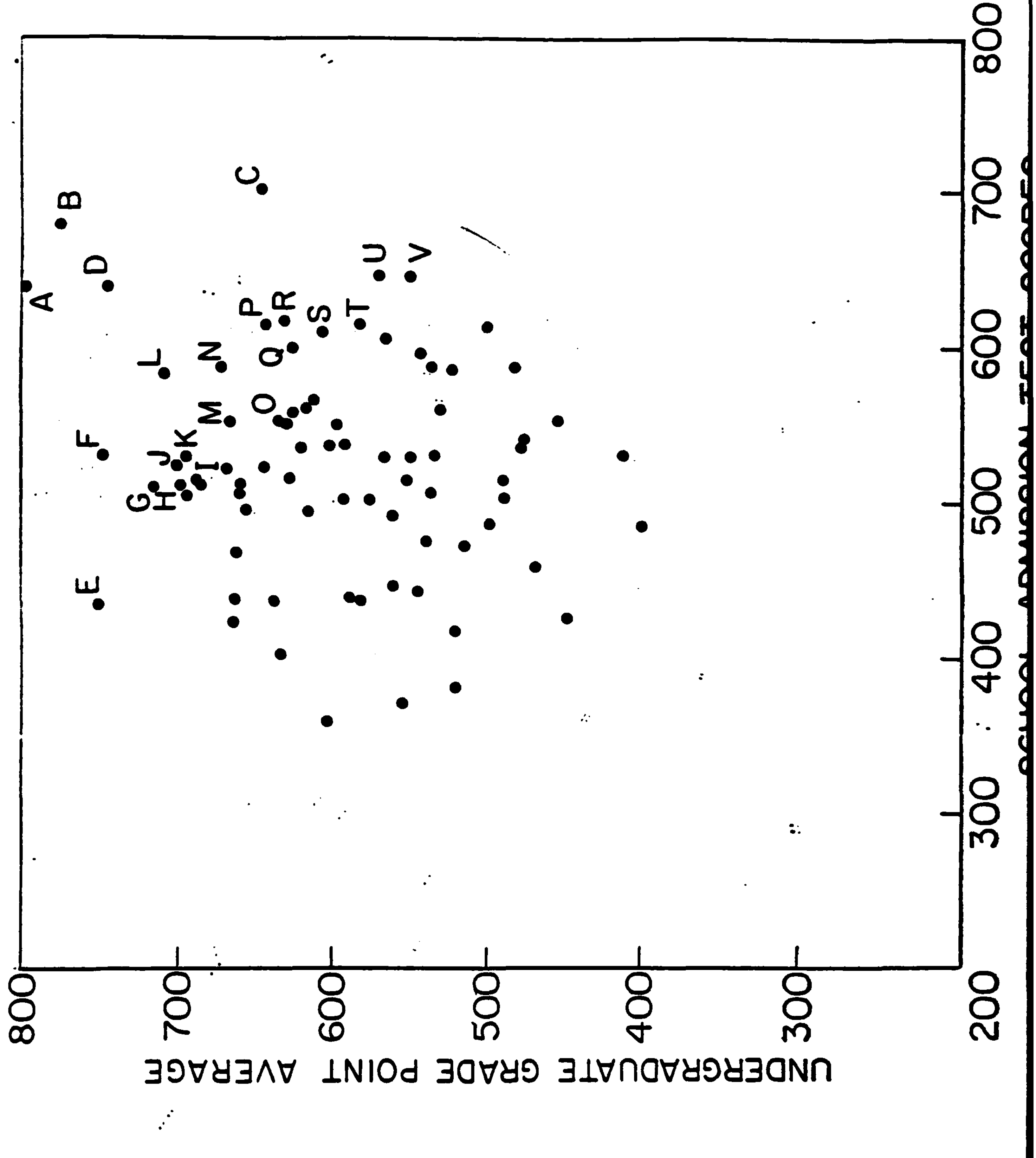
In other words, in evaluating the effects of one decision rule (such as equal weighting combined with a cut-off on aggregate score) as compared with another (such as a regression coefficient combined with a similar cut-off), one should ignore those alternatives not in actual contention. This notion, translated into the technical language of ordinary dominance and Pareto frontiers, is the essential idea first noted by Seaver, and later by Newman and McClelland. But instances like that of graduate admissions, and many others, have a slightly more complicated structure than that considered by these researchers. The task is, not to pick the best one out of $\underline{n}$, but to pick the best $\underline{k}$ out of $\underline{n}$. The difference between the two cases is that in the pick-$\underline{k}$ case, there are not only inevitable losers but also inevitable winners. If Seaver's argument for excluding the inevitable losers from calculations concerning composite correlations (and inter-attribute correlations) is appropriate, an argument for exlcuding the inevitable winners would seem equally appropriate, for exactly the same reason--they are irrelevant elements of an ill-defined population.

By now it should be apparent that the central theme of this paper is that, given a set of evaluative criteria, the alternatives being evaluated for any decision problem can be divided into three classes: sure winners, sure losers, and contenders. (Of course, any of these sets may be empty.) First, we will motivate the formal definitions and results concerning winners, losers, and contenders with a simple two-dimensional example. All ideas easily generalize to any finite number of dimensions, substituting hypercubes for squares, hyperplanes for lines, and so on.

II. A Two-Dimensional Example

Since graduate student admissions policies have been so familiar in the equal weights literature, we use them also. Suppose that a set of 79 applicant folders is being considered by a law school admissions committee. The committee agrees to base all decisions on exactly two dimensions: undergraduate grade point average (UGPA) and law school admissions test (LSAT) scores. On each dimension considered separately, the committee agrees that higher values are preferred to lower. (A more technical discussion in an n-dimensional context would have to assume that, for all possible subsets of dimensions, increase in values on any or all of the dimensions cannot decrease aggregate attractiveness--a condition naturally fulfilled in many decision problems.) The 79 applicants, plotted in Figure 1 (taken from an ETS technical report, Rubin, Note 4), yield a Pearson product-moment correlation of .168. If equal weights are used in place of

FIGURE 1


UNDERGRADUATE GRADE POINT AVERAGE
800
700
600
500
400
300
200
300
400
500
600
700
800
A
B
C
D
E
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Q
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U
V

some other "true" set of weights, the worst possible correlation between composite evaluations is .82 (when the "true" weights are extreme--0, 1). Clearly, this is an example for which equal weights would seem appropriate.

We digress for a moment to note the misleading nature of that .168 correlation. It is based only on the folders in the hands of the admissions committee--a strangely defined population. Had students who were prescreened out, or who chose not to apply to law school, or who confined their applications to more prestigious or more convenient schools, been included, the correlation would almost certainly have been higher. The correlation is likely to be sensitive to many non-population characteristics, such as the school's recruiting effort, the various obstacles that are placed in the way of completing the folder, and informal pre-screening procedures. Do the folders represent a sample from a defined universe? In the absence of a defined population and a suitably random set of rules for sampling from it, any correlation or regression coefficient is only a statistic, descriptive of the data in hand. And in decision contexts, both well-defined populations and random samples are rare, if they occur at all. Consequently, considered as estimates of population parameters, such numbers are doubtful--even if in actual experience they replicate from year to year. (That simply means that non-random processes leading to inclusion of elements in a choice set have changed slowly, but, of course, there can never be a guarantee that they will remain stable next time.) We think this fact presents major difficulties in the application of the Brunswikian idea of representativeness to decision contexts.[1] How can one specify what is representative, or what is being represented? But that is not the point of this paper.

Consider first the case in which the admissions committee planned to admit only one student. There are no sure winners. There are, however, exactly three contenders (A,B,C). All others are sure losers, since they are dominated. The correlation between UGPA and LSAT among the three contenders is -.90, a most inappropriate case for equal weights. The argument we wish to make is nothing more than a generalization of this idea.

Suppose the admissions committee plans to admit eight applicants (roughly 10%). Now there are some sure winners, some sure losers, and some contenders; our task is to identify them. An applicant, X, is a sure winner if and only if there are fewer than k applicants, y, in the total choice set for which $\underline{u}(y) \geq \underline{u}(X)$, for any utility function $\underline{u}$. Thus, if fewer than k applicants are better than X on either UGPA or LSAT, X is a sure winner. In the present sample, for k equal to eight, there are exactly three sure winners (A, B, D). X is a sure loser if and only if there are at least k applicants, y, in the total choice set for which $\underline{u}(y) > \underline{u}(X)$, for any $\underline{u}$. That is, if k or more applicants dominate X (i.e. better on both UGPA and LSAT), X is a sure loser. There are 57 sure losers. Any point which is not a sure winner or a sure loser is a general contender. Whether contenders are ultimately chosen or not depends upon the form and parameter values of the utility function, $\underline{u}$. The 19 contenders, labelled E through V, yield a correlation of -.78 between UGPA and LSAT. As in the case of k equal to one, applicant evaluations that use equal weights may not correlate highly with those in which UGPA and LSAT are weighted differentially.

By placing restrictions on the form of the utility function, u, the set of contender applicants may be further reduced in size. For example, suppose that the admissions committee decided that the evaluations should be an additive function of UGPA and LSAT. Graphically, any choice of u can now be represented by a line with negative slope, the magnitude of which is a function of the weights attached to UGPA and LSAT. (Equal weights would produce a line with slope -1.) An applicant X is an additive sure winner if every negatively sloped line, l, passing through X yields fewer than k points above and to the right of l (including other points on l and sure winners). Additive sure losers are those contender applicants for which every line l through X yields k or more points above and to the right of l (again including other points on l and sure winners) The remaining points are additive general conten

It is not possible to perform the required calculations for every negatively sloped line--since there are uncountably many of them. However, for purposes of counting applicants above and to the right of the lines, only those lines passing through another applicant (also a contender) need be considered. Thus, points must be counted for only 18 different lines for each of the 19 contenders (342 total). Of the 19 general contenders, 8 are additive sure losers, while the remaining 11 are additive contenders. Although no additional sure winners were obtained, this is not necessarily the case for all problems. Among the additive contender applicants, UGPA and LSAT are correlated -.79. Again, this case is not well suited for equal weighting.

Initially, the decision problem was to select 8 entering students for a law school from a set of 79 applicants. Basing the selections upon (monotonic) functions of UGPA and LSAT reduced the problem to one with only 19 contenders for 5 slots. Basing applicant evaluations upon a weighted average of UGPA and LSAT reduced the set of contenders to only 11. Thus, the fates of at least 76% of the applicants were determined independently of weights; over a third of the admissions made did not depend on weights. However, among the 11 applicants to which the decision rule was relevant, the ordering (and subsequent selection) was highly sensitive to the weights.

III. A Simulation

Having motivated the central ideas necessary for an analysis of the pick k out of n problem, the remainder of the paper is devoted to a Monte Carlo investigation. Using the pivot procedure of matrix factoring, 160 m-tuples were generated from a multivariate normal distribution (see Newman, Seaver, & Edwards,Note 1). A breakdown of the Monte Carlo design is given in Table 1. The covariance structure of the multivariate normal distributions of alternatives maintained equal expected correlations among all pairs of attributes (either -.2. 0.0. 0.5. or 0.9). The proportion of alternatives to be chosen, k, was varied for values less than 50%, since all dependent variables were expected to be symmetric about k = 50%. (Preliminary runs verified that this was indeed true.) Data for all ten of the within-sample factor cells were obtained from each of the 160 generated samples. For the various combinations of factor levels, each of five samples were partioned into three groups: (additive) sure winners, (additive) sure losers, and (additive) general contenders. The number of alternatives in each of these three sets, as well as

TABLE 1

Monte Carlo Design

Factor	No. of Levels	Levels
No. of points (density)	2	50, 100 points
No. of dimensions	4	2, 3, 4, 5 dimensions
r_{ij} in initial interattribute correlation matrix	4	-0.2, 0.0, 0.5, 0.9
Total No. of Between Cells	32	
Proportion of points chosen	5	2, 10, 20, 30, 40 %
Utility function restrictions	2	None, Additive
Total No. of Cells	320	
Repetitions	5	
Total No. of Cases	1600	

the average inter-attribute correlation among the contenders, was computed.

The problem of obtaining the additive partitioning is solvable as a non-linear programming problem (non-linear and non-continuous objective function, linear constraints). The representation of weight vectors as lines in two dimensions was generalized to hyperplanes in m dimensions. Thus, for each of the c general contenders, $\binom{c-1}{m-1}$ different weight vectors must be solved for and considered. Thus, for each repetition, c $\cdot$ $\binom{c-1}{m-1}$ matrices of size m x m must be inverted. Table 2 displays this number for values of m = 2, 4, 8, and c = 10, 50, 100. The computing time and costs for arithmetic of this volume are phenomenal, if not absolutely prohibitive. Thus, another solution for the additive case was sought.

Fishburn (1965) proved a dominance theorem based on Abel's well known summation identity. The theorem provides a method for eliminating strategies (from the ordinary outcome by event pay-off matrix) based on rank order information about the event probabilities. Of course, since the mathematical structure of the MAUM problem is identical to that of ordinary decision theory, the same theorem applies to weights. Thus, for any given rank ordering of the weights, one can partition the alternatives into winners, losers, and contenders.[2] If one performs these calculations for every possible rank ordering, then the intersection of all of the sets of sure winners will be the set of additive sure winners. The intersection of all sets of sure losers will be the set of additive sure losers. Of course, the remaining alternatives are additive general contenders.

TABLE 2

Number of $\underline{m}$ x $\underline{m}$ Matrices to be Inverted
For $\underline{m}$ Dimensions and $\underline{c}$ General Contenders

No. of Contenders (c)	No. of Dimensions (m)		
	2	4	8
10	90	840	360
50	2450	9.2×10^5	4.3×10^9
100	9900	1.6×10^7	7.3×10^{13}

Since the results are essentially identical for the two levels of total number of points (50 and 100), all of the results will be reported for 100 points only. The proportion of sure winners is displayed in Figures 2 and 3. Each data point represents five repetitions. The maximum possible value for this proportion is the k%, represented by the long-short dashed line. The main effects here are for initial attribute intercorrelation and number of dimensions. As the attributes become more highly correlated, the percentage of sure winners increases dramatically. This result is attenuated for higher numbers of dimensions. The increase in sure winners caused by the additive utility restriction is quite modest. Also, the proportion of sure winners seems to increase roughly linearly with the proportion to be chosen (k).

Proportions of general contenders are displayed in Figures 4 and 5. These results are complementary with those for sure winners: higher numbers of dimensions mean more contenders, and higher initial correlations among attributes signal fewer contenders. We suspect that the contenders peak at k = 50% (roughly the same as k = 40%) because of the multivariate normal distributions used. Had the distributions of alternatives been skewed, the proportion of contenders would not have been symmetric about k = 50%, and the k = 50% peak would not have resulted. There seems to be a somewhat larger effect for the additive restriction: if one can assume that u is additive, then substantially fewer alternatives need be considered at all.

The results for the average intercorrelation among pairs of attributes for general contenders are presented in Figures 6 and 7.

FIGURE 2

PROPORTIONS OF SURE WINNERS – Unrestricted u

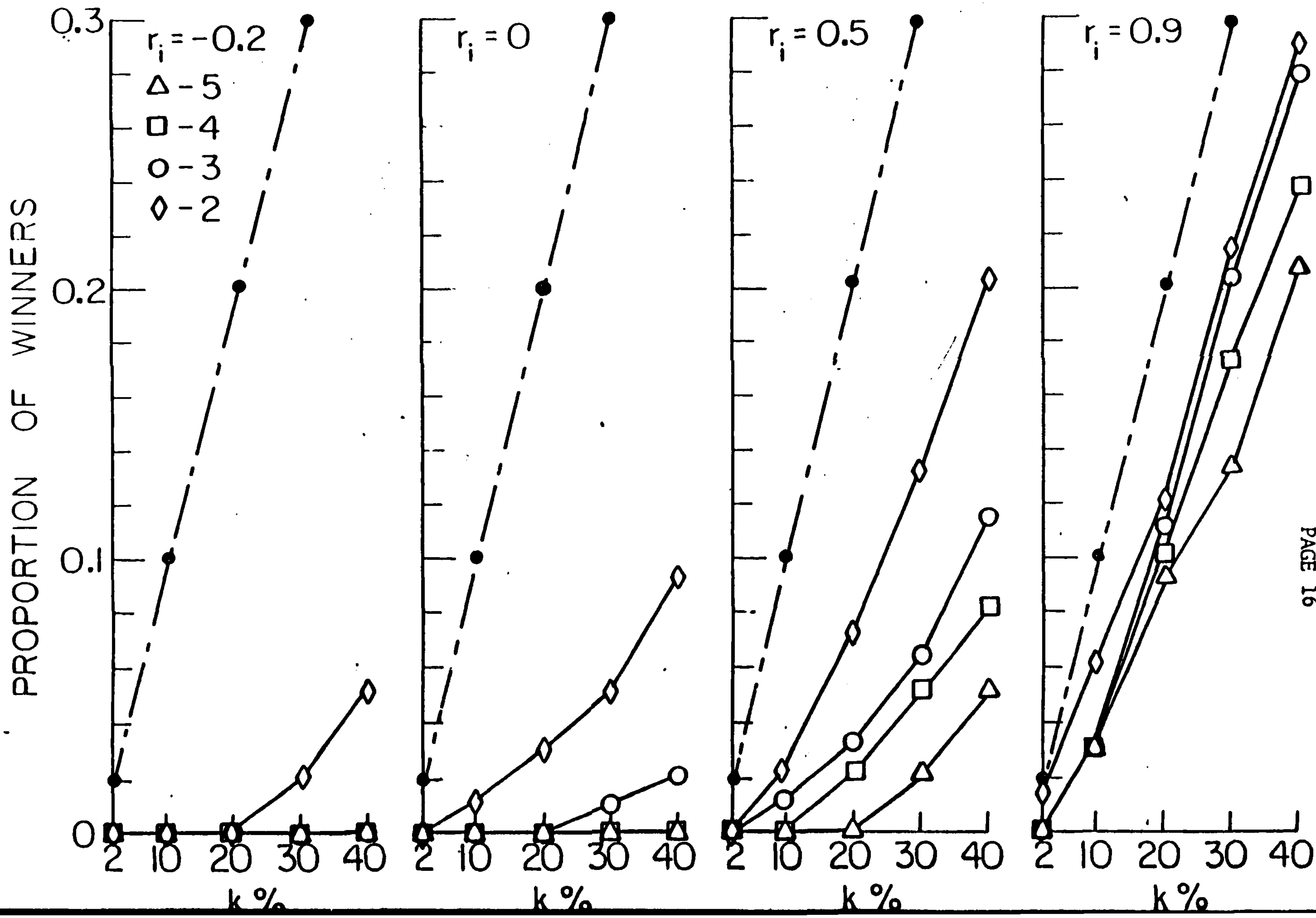
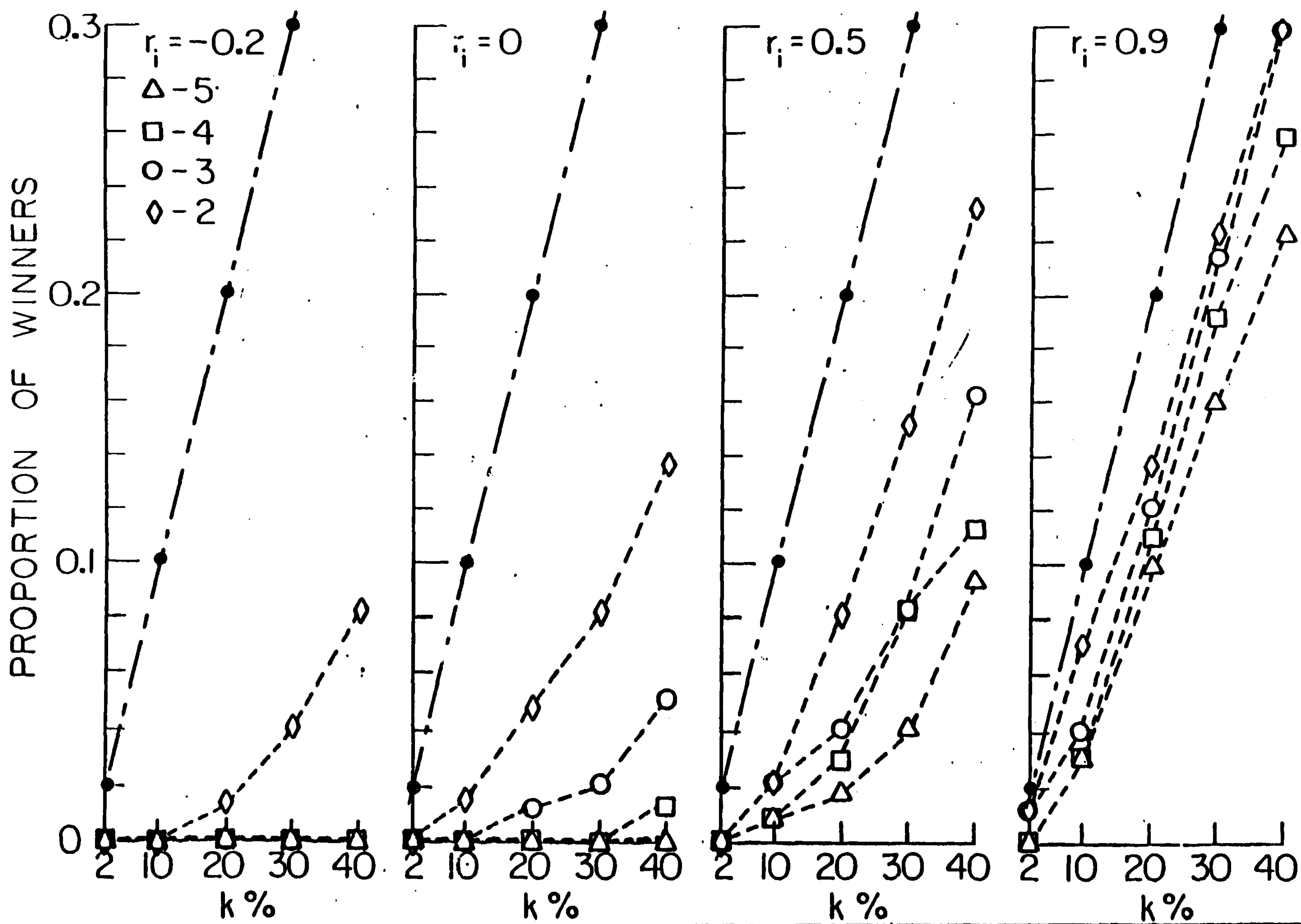
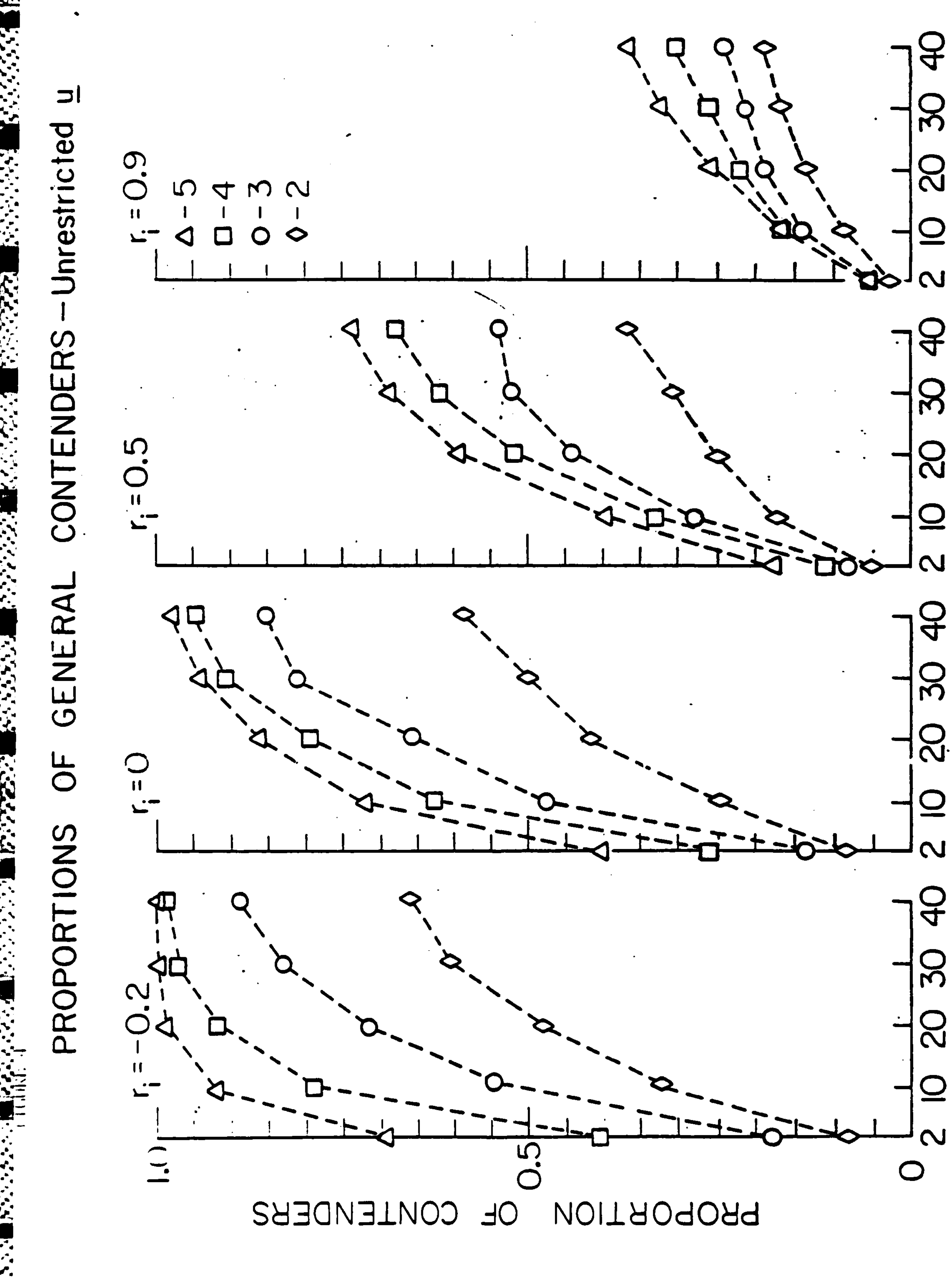


FIGURE 3

PROPORTIONS OF SURE WINNERS — Additive u





PROPORTIONS OF GENERAL CONTENDERS - Unrestricted u
PROPORTION OF CONTENDERS
1.0
0.5
0
$r_i = -0.2$
$r_i = 0$
$r_i = 0.5$
$r_i = 0.9$
△ - 5
□ - 4
○ - 3
◇ - 2
2 10 20 30 40
k %

FIGURE 5

PROPORTIONS OF GENERAL CONTENDERS — Additive u

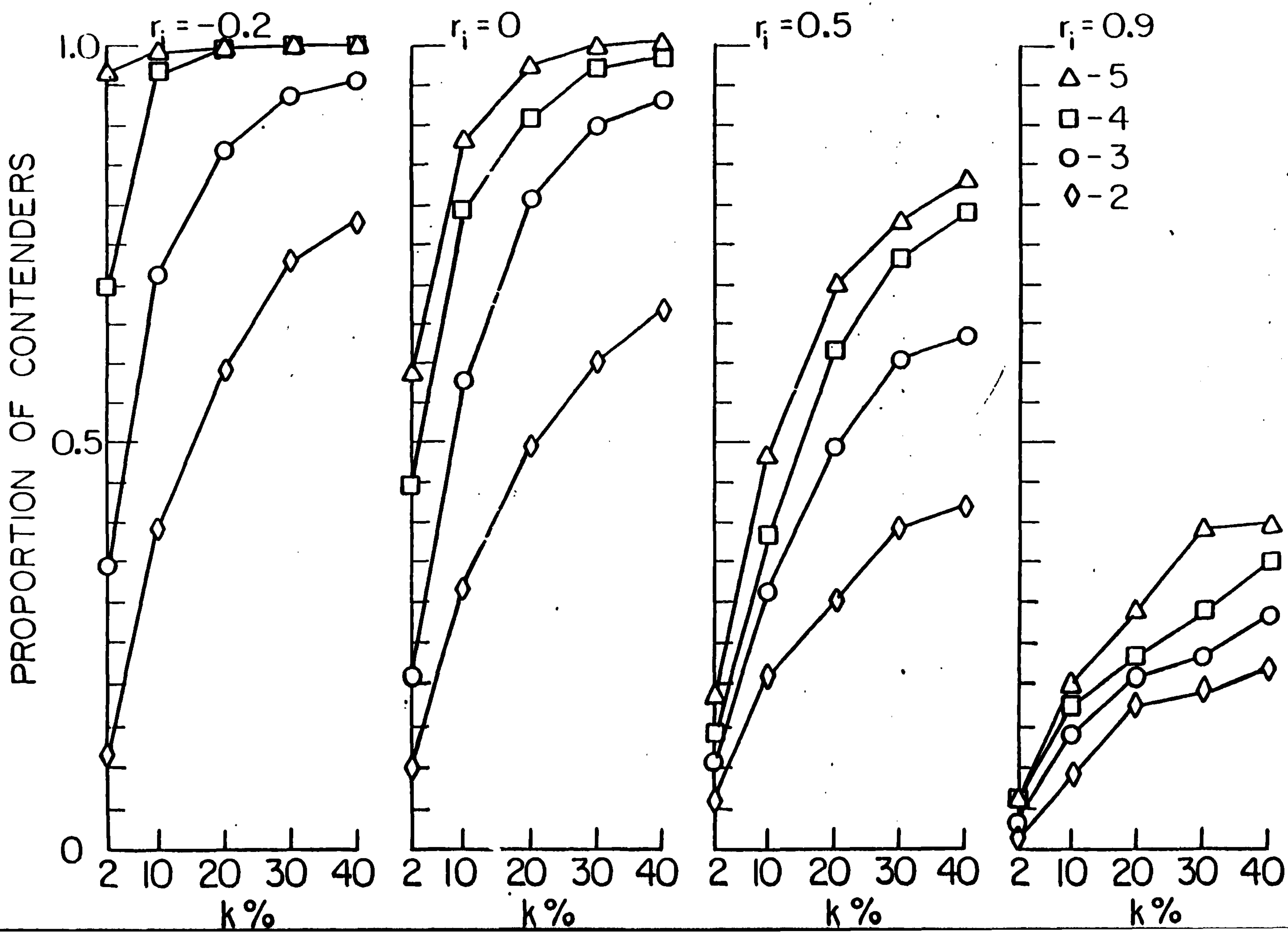


FIGURE 6

AVERAGE ATTRIBUTE INTERCORRELATIONS AMONG GENERAL CONTENDERS — Unrestricted u

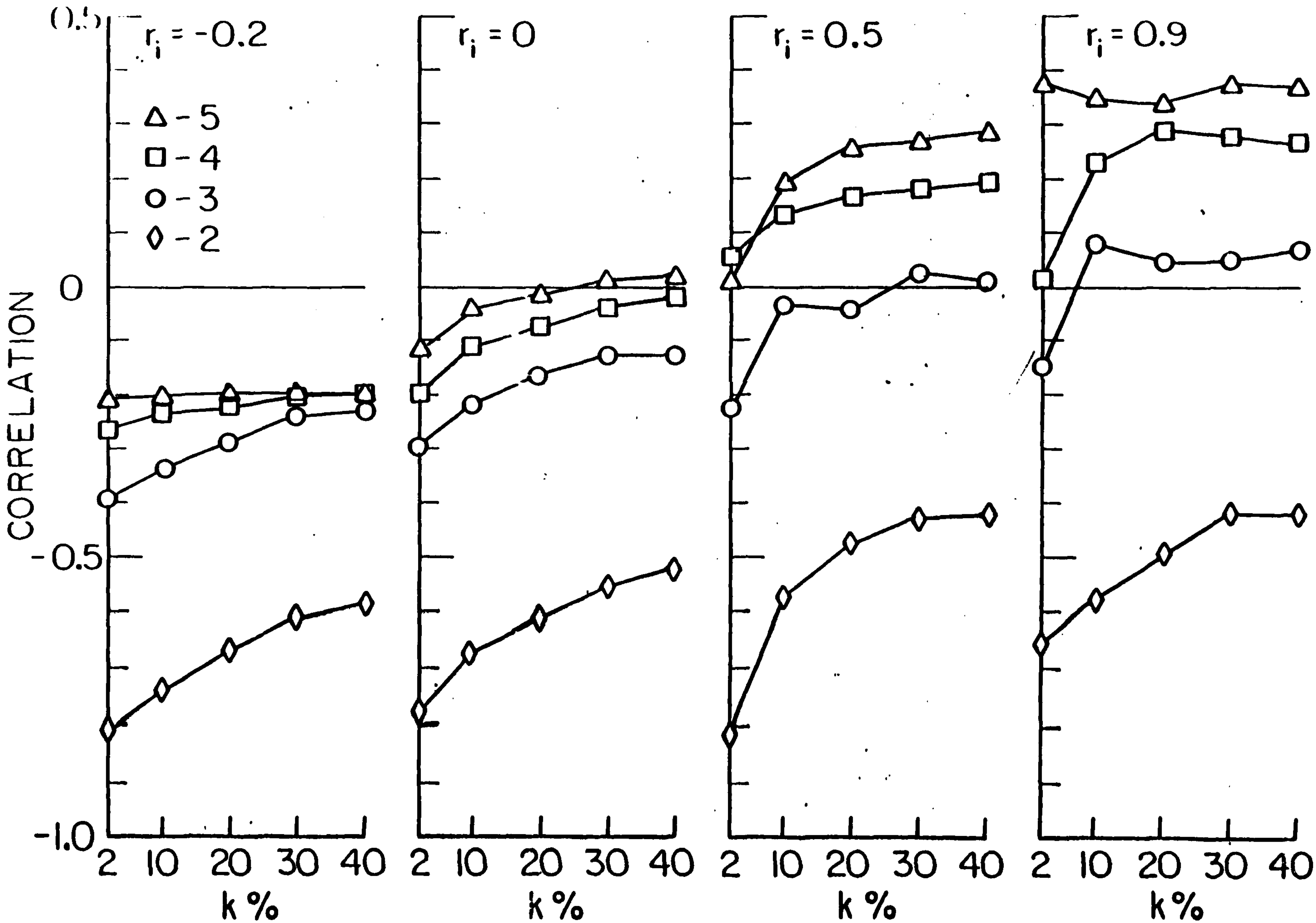
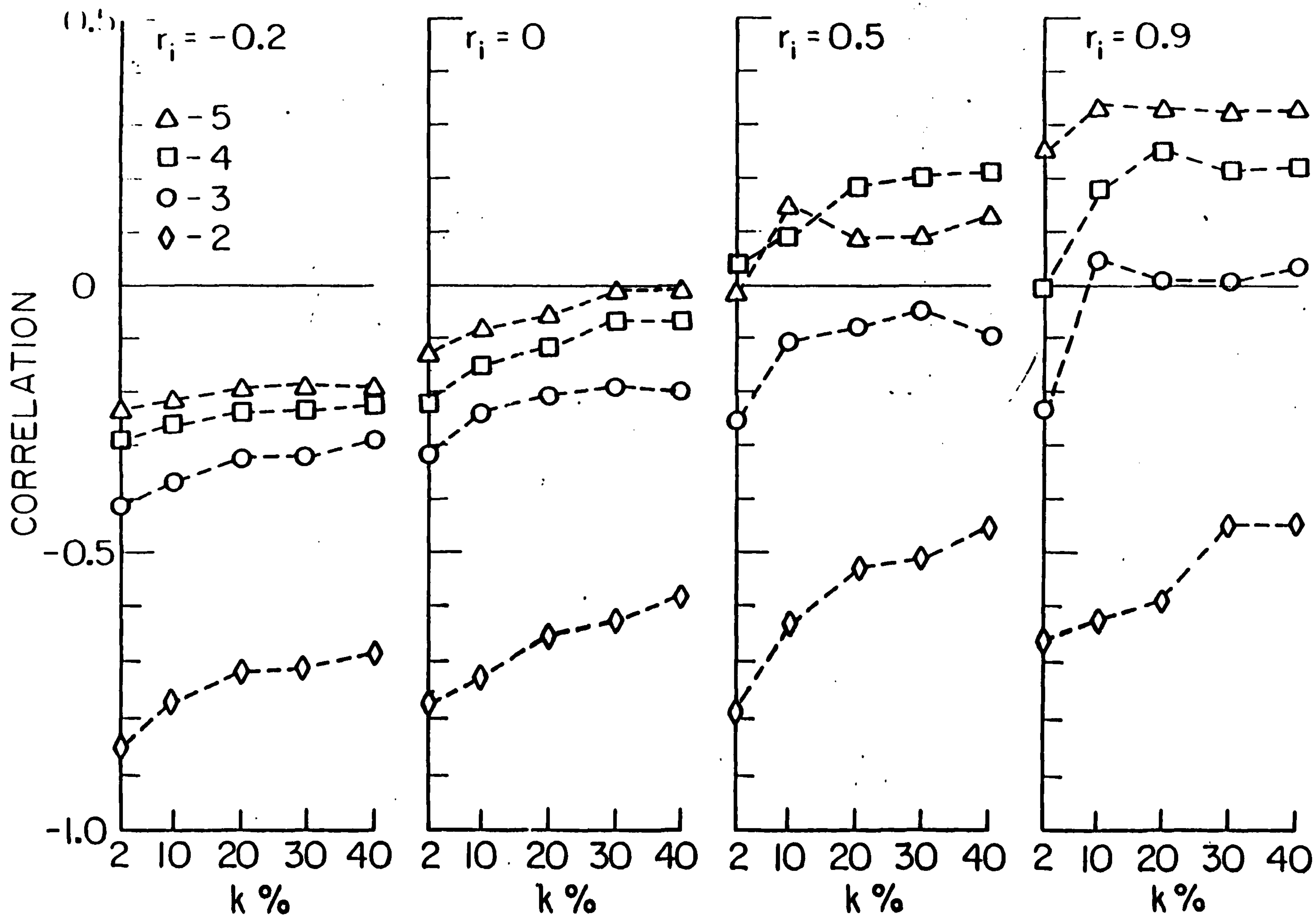


FIGURE 7

AVERAGE ATTRIBUTE INTERCORRELATIONS AMONG GENERAL CONTENDERS — Additive u



The most profound effect here is for number of dimensions. The average correlation rises rapidly as the number of dimensions increases. The average attribute intercorrelations rise monotonically as a function of the initial correlation among attributes. Thus, high initial correlations produce both fewer contenders and higher attribute correlations among the contenders. The effects of $\underline{k}$% and the additive form restriction are not very great. In general, the additive general contenders produce more negative attribute intercorrelations than do the non-$\underline{u}$-restricted general contenders. The attribute intercorrelations also increase as a function of $\underline{k}$%. It is noteworthy that the average attribute intercorrelations for the pick 2%, $\underline{r}_i = -.20$ case (far left panel) are nearly equal to the smallest values possible. For 2, 3, 4, and 5, dimensions, the least possible correlations are -1, -.5, -.33, and -.25, compared to the values -.85, -.40, -26, and -.21 obtained. The inter-attribute correlations among contenders for the $\underline{k}$ = 2%, $\underline{r}_i$ = 0 case are in good agreement with the ones McClelland published in the pick-one case using hypothetical "typical" Pareto frontiers.

IV. Conclusions

Our simulation produced two main results. The first is the triviality of many decisions in a multiattribute decision context where the problem is to accept $\underline{k}$ out of $\underline{n}$ candidates. We showed that two conditions guarantee a large proportion of sure winners and sure losers: if attributes are positively correlated and if the number of attributes is relatively small. Correspondingly, the proportion of contenders decreases with increasing attribute intercorrelations and decreasing number of attributes.

We had speculated that the reduction of contenders would severely depress intercorrelations between attributes. Indeed, this is the second result of our simulation. In the two attribute case the attribute intercorrelation becomes highly negative, independent of the initial correlation in the whole set, independent of the acceptance ratio (k/n), and independent of the restrictions on the utility function. This is, of course, a generalization of McClelland's findings. The depression effect is maintained for higher number of attributes, although the average correlation among attributes is less highly negative, and in some cases (i.e. when the original correlations are .9) even slightly positive.

The increase in the average correlation for higher numbers of attributes was expected. It is well known that the constraints imposed on a correlation matrix create a lower bound on the average intercorrelation, which tends to go to zero for large n. On the surface this seems to contradict our argument for the prevalence of negative correlations among attributes. However, near zero average correlations can be obtained by high correlations with opposite signs. While our simulation does not provide any direct evidence of such a correlational structure, the real world often does. Consider, for example, the attributes characterizing cars, which are usually grouped into "cost" and "quality." The market place and scarce resources produce negative correlations between cost and quality. Lower level attributes are probably positively correlated within each of these two general dimensions and negatively across, thus producing exactly the kind of opposite sign correlation matrix which can cause weights to be sensitive. We believe that such correlational structures are relatively common among contender sets. The underlying reason is the hierarical organization of values in which the highest level objectives are usually in conflict.

The arguments favoring equal weights rely on zero or positive inter-attribute correlations. Our results suggest that such positive correlation structures may be more the exception than the rule in multiattribute decisions. We conclude therefore that the equal weights argument cannot easily be generalized from its original domain (prediction, with usually high positive intercorrelations among predictors) to multiattribute decision making (with often severly depressed or negative attribute intercorrelations).

From the depressed and negative inter-attribute correlations we can further conclude that the correlations among composites between equal and non-equal weighting schemes will be much lower when restricting the set of contenders than when applied to the whole set. Thus, in generalization of previous results, we believe that equal weights will often do poorly in multiattribute decision making when evaluated on the basis of correlations among composites.

To make a final evaluation of equal weights as an approximation rule in MAUM, we would have to go one step further, and analyse losses in utility. It was exactly this spirit of losses in utility which led us to examine contenders in the first place, since sure winners and sure losers can never lead to a loss in utility. McClelland analyzed a special form of utility loss in the two attribute case and concluded that equal weights would do poorly if the Pareto frontiers close to a straight line. V. Winterfeldt and Edwards (Note 5) studied utility losses at the Pareto frontier for a higher number of dimensions and argued that, in general, differential weighting will not lead to large utility losses. No one has examined utility losses for the _k_ of _n_ problem yet.

FOOTNOTES

[1]Egon Brunswik (Hammond, 1966) first called attention to the importance of ecological validity, and consequently to the importance of correlations among cues. He correctly pointed out that orthogonal sets of cues exist only in the psychological laboratory. He also argued that the set of correlations among cues should in some sense represent "the causal texture of the environment." He interpreted this is to mean that they should be sampled in some sense representatively from the set of environmental inter-cue correlations. While his methods were informal, some case can be made that he did in some sense attempt to accomplish this, although of course, he could not define the universe of inter-cue correlations, and so could not use formal sampling methods.

[2]For technical details of the procedure, see Fishburn (1965) and Barron (1973).

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Egon Brunswik first called attention to the importance of ecological validity and consequently to the importance of correlations among cues. Cue intercorrelations and value independence are central to weights in linear models used for prediction. Dawes and Corrigan, Einhorn and Hogarth, and Wainer have argued that equal weights are usually better than least-squares regression weights. Newman, Seaver, Edwards and McClelland have all shown that if cue intercorrelations are positive or zero, equal weighting works very well, but if cue

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intercorrelations are negative, equal weighting can lead to serious errors of prediction.

Most predictions are intended as a basis for decision making. The point of this paper is that prediction and decision require different methods. Equal weights, while often useful for prediction, are less useful for decision making.

The action options available in any decision problem fall into three classes: sure winners, sure losers, and contenders. Sure winners and sure losers are defined by dominance; accepting sure winners and rejecting sure losers is trivial. Good decision rules should discriminate well among contenders.

In the familiar pick-1 decision problem, options on the Pareto frontier (i.e. undominated options) almost always show negative correlations among attributes. Such negative correlations make equal weights inappropriate.

This paper extends that result to the case in which a decision maker must pick k options out of n. In this case, the set of sure winners is usually not empty. It develops general procedures for identifying the set of contenders, given the options, k, and n. This set is a generalized Pareto frontier, of which the traditional kind is a special case. Simulation show that attribute intercorrelations among contenders are substantially depressed and typically negative, even if the intercorrelations in the whole set are positive. Such negative correlations among contenders strongly question the usefulness of equal weights for decision making.

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